AIAA JOURNAL
Vol. 35, No. 1, January 1997

Automatic Control of Laminar Boundary-Layer Transition

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Interest has recently been renewed in the use of distributed suction for the production of laminar flow over substantial areas of the surface of aircraft wings and engine nacelles. Suction may be most efficiently applied by using a number of independently controllable panels through which fluid is withdrawn. The need to determine the distribution of suction flow rates that results in a given streamwise location of boundary-layer transition with minimum power consumed in providing suction gives rise to a nonlinearly constrained optimization problem. A gradient descent algorithm is shown to be successful in both experimental and numerical studies in determining the optimal suction distribution. We present results of experiments performed on a flat plate in a wind tunnel with two suction panels and an optional pressure gradient. We show that the system successfully maintains the laminar-turbulent transition at a given point and minimizes the amount of suction power required to do so.

Nomenclature

d = distance from origin to optimum point
e = transition position error
g = pump energy cost
J = Lagrangian function
k = time step index
M = number of microphones
N = number of suction panels
$\langle p \rangle$ = rms average pressure
R = radius of curvature of the $e = 0$ contour
Re_y = Reynolds number based on transition position
r = desired transition position
T = averaging time for rms measurements
U_∞ = mean flow velocity
u_i = ith suction velocity or pump voltage
$\boldsymbol{u}*$ = optimum suction vector
x = distance from leading edge to microphone
y = distance from leading edge to transition
α = error reduction parameter
λ = Lagrange multiplier
μ = step size parameter

Introduction

IN this paper we present a new approach to the control of boundary-layer flow. The work described is motivated by the prospect of substantial reduction in skin-friction drag produced by the maintenance of laminar flow on the wings and nacelles of commercial aircraft. It has long been known that relatively small amounts of suction through the surface over which a boundary layer develops can stabilize the flow and delay transition to turbulence, which has been shown to provide a beneficial stabilizing effect under flight conditions,[1] especially when used in a hybrid laminar flow control scheme. In this case, suction is applied close to the leading edge of a wing or nacelle whose cross section is also shaped to produce an initial region of favorable pressure gradient. In the flight experiments reported by Maddallon et al.,[1] suction was applied at the leading edge of the wing of a Jet Star by using a suction compartment consisting of 15 individual suction panels. The suction flow rates through these panels were individually adjusted to derive the desired distribution of suction. The results clearly illustrated the sensitivity of the required suction distribution to the flow conditions, particularly the presence of cross-flow instabilities in the leading-edge region. However, although systematic variations were made in the distribution of suction applied, there appeared to be no method available for the determination of an optimal distribution of suction. Similarly, in wind-tunnel studies reported by Mullender and Poll[2] the suction flow rates associated with eight individual suction panels were manually adjusted to deduce the suction distribution necessary to just maintain laminar flow over a given chordwise region.

Hitherto it has been the objective of any laminar flow control scheme to minimize the total net drag on the aircraft. It is also clear that the application of suction itself consumes power, and it is quite possible that under some flight conditions the maintenance of laminar flow over a sufficiently large area of wing surface may prove detrimental to the total energy balance. In fact, an examination of the drag balance and the energy balance for a single suction panel beneath a flat plate boundary layer shows that there is an optimal transition location (and associated suction rate) that minimizes the total power. This transition location is, however, sensitive to both mean flow speed and the disturbance environment.[3,4]

These factors suggest that to achieve the full benefits of laminar flow control it will be necessary to monitor the extent of laminar flow, the flight conditions, and the power consumed in providing suction. Furthermore, it may be desirable to automatically adjust the suction distribution in response to such measurements in order to minimize energy expenditure. In this paper, we shall deal with the determination of the optimal distribution of suction under given flow conditions; i.e., we seek to determine an optimal distribution of suction that maintains transition in some desired position. This allows us to convert the multidimensional problem of minimizing total energy consumption by adjusting many suction flow rates to the one-dimensional problem of doing so by adjusting the transition position.

In the following, we present an algorithm that is well suited to this problem. It has its origins in the gradient projection method developed by Rosen,[5,6] which was modified, in the case of a linear constraint, by Frost[7] for use in adaptive array processing. More recently Frost's approach has been used as the basis for adaptive algorithms for underdetermined active control problems by Elliott and Rex.[8] Here Frost's approach is modified to deal with nonlinear constraint functions. The algorithm derived exhibits some properties that are highly desirable within the current context. The control variables (suction flow rates) are at first rapidly adjusted to ensure that the constraint is satisfied (the desired position of transition is reached) and then subsequently adjusted to ensure the minimization of the cost function (the sum of squared suction flow rates).

We then report the results of an experimental implementation of this algorithm on a flat plate with two suction panels mounted in a wind tunnel whose walls can be shaped to induce a nonzero pressure gradient along the plate. Microphones mounted in the surface of the plate were used to detect the transition position to close the control loop. We show that the algorithm successfully converges to an optimal suction distribution in a reasonable time and that the stability

Received Nov. 14, 1995; revision received May 16, 1996; accepted for publication Aug. 21, 1996; also published in *AIAA Journal on Disc*, Volume 2, Number 1.

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conditions we have derived enable us to choose a suitable control gain. These results, coupled with the theory presented herein, form the groundwork for a system whereby an aircraft automatically maximizes its energy efficiency as it flies—a smart wing. A more immediate application of the algorithm is likely to be in wind-tunnel testing, where it is necessary to determine the flow rates associated with a number of suction panels in a laminar flow control scheme. It is quite possible, however, that the algorithm could be used under flight conditions; for example, it could be applied to a test aircraft to develop a library of suction distributions for use in different flight conditions, which could then be used by other aircraft of the same model.

Optimization Problem

The problem to be dealt with is illustrated in Fig. 1. The input to the system is the vector of suction flow rates $\boldsymbol{u}^T = (u_1, u_2, \ldots, u_N)$, where u_i is the flow rate on the ith panel. The output of the system is given by $y(\boldsymbol{u})$, the position of transition. The typical form of this function for a system with two suction panels is sketched as a contour plot in Fig. 2 and has been established both in these experiments and in the numerical studies presented by Hackenberg[9] and Hackenberg et al.[10]

We require the system to maintain the value of y at some desired position given by r, thus defining an error function

$$e(\boldsymbol{u}) = y(\boldsymbol{u}) - r \tag{1}$$

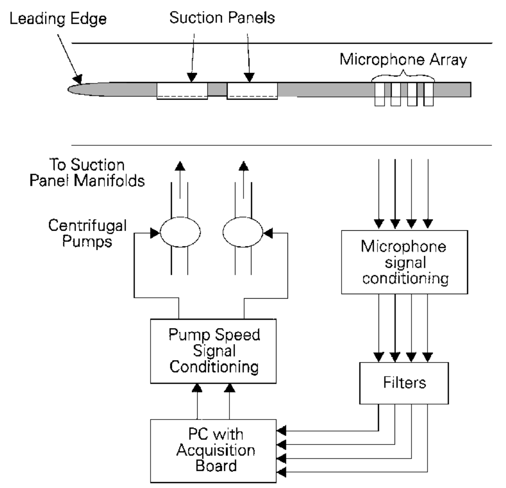

Fig. 1 Schematic diagram of the boundary-layer control system.

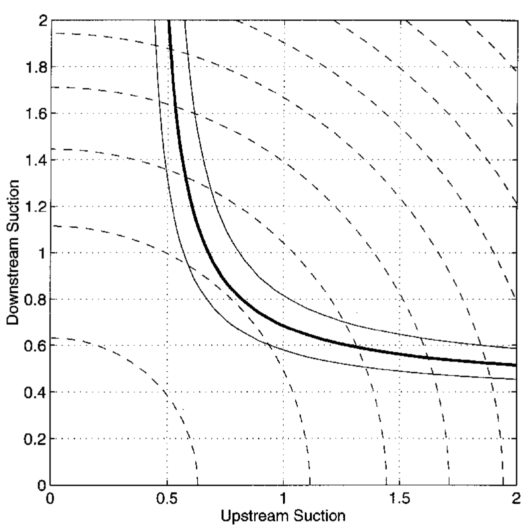

Fig. 2 Idealized sketch of the dependence of transition position y on the suction flow rates associated with two suction panels beneath a flat plate boundary layer, showing contours for three particular values of y with $y = r$ in bold.

The zero-error contour for some arbitrary r is shown in bold in Fig. 2. We wish to find the optimal suction flow rates $\boldsymbol{u}*$ that satisfy the constraint $e(\boldsymbol{u}) = 0$ while consuming the least energy. This energy cost will be modeled by the quadratic function

$$g(\boldsymbol{u}) = \tfrac{1}{2}\boldsymbol{u}^T \boldsymbol{u} \tag{2}$$

Contours of constant values of $g(\boldsymbol{u})$ in this case are also shown by the dotted lines in Fig. 2. The choice of this cost function for minimization is somewhat arbitrary—it could be argued that the power consumed includes terms that are linear and cubic in the suction flow rate[3]; however, the analysis presented next is sufficiently general to allow a different choice of $g(\boldsymbol{u})$ if needed. The problem that we wish to solve can be stated generally as

$$\text{minimize}\ g(\boldsymbol{u}) \ \text{subject to} \ e(\boldsymbol{u}) = 0$$

This type of problem has been widely studied in the literature of optimization, and, for example, Gill et al.[11] give necessary conditions for such a constrained minimum to exist at a point $\boldsymbol{u}*$, namely, that the vectors $\nabla e(\boldsymbol{u}*)$ and $\nabla g(\boldsymbol{u}*)$ must be parallel, so that there must exist λ such that

$$\nabla g(\boldsymbol{u}*) + \lambda \nabla e(\boldsymbol{u}*) = 0 \tag{3}$$

This defines a stationary point of the so-called Lagrangian function

$$J = g(\boldsymbol{u}) + \lambda e(\boldsymbol{u}) \tag{4}$$

where λ is a Lagrange multiplier whose value has to be determined.

If, as turns out to be the case for many geometries, the stationary point of J is a strong local minimum, then we can find the solution of the constrained optimization problem by solving an unconstrained optimization problem, that of minimizing J, when λ has been correctly estimated. The following algorithm, first presented by Nelson and Rioual,[12] will achieve this goal by updating the suction flow rates according to

$$\boldsymbol{u}_{k+1} = \boldsymbol{u}_k - \mu \nabla g_k + \left[\frac{\mu \nabla g_k^T \nabla e_k - (1 - \alpha) e_k}{\nabla e_k^T \nabla e_k} \right] \nabla e_k \tag{5}$$

Full details of the derivation are given in Ref. 12; briefly the suctions are updated so as to perform steepest descent optimization on J, i.e.,

$$\boldsymbol{u}_{k+1} = \boldsymbol{u}_k - \mu \nabla J_k \tag{6}$$

while requiring the error to reduce by a constant factor so that

$$e_{k+1} = \alpha e_k \tag{7}$$

where $0 \leq \alpha < 1$. This leads to an estimate of λ at the kth iteration of

$$\lambda_k = \frac{-\mu \nabla g_k^T \nabla e_k + (1 - \alpha) e_k}{\mu \nabla e_k^T \nabla e_k} \tag{8}$$

which gives us the algorithm (5), which is suitable for adaptive solution of nonlinearly constrained optimization problems.

Simulations using known functions for e and g show relatively rapid progress to the zero-error contour, followed by steady progress along the zero-error contour towards the constrained optimum. The term in Eq. (5) that is directly proportional to the error e_k is the one that is mainly responsible for producing this rapid convergence to the zero-error contour.

Saunois et al.[13] have examined the stability of the algorithm by treating it as an iterated map and examining its dynamics in the region of a fixed point and found the following condition on μ:

$$\mu < \frac{2}{1 + d/R} \tag{9}$$

In the case of a linear constraint, $d/R = 0$, and $\mu = 2$ represents an upper bound on the values of μ that can be used. When the constraint has a high curvature in the region of the fixed point, then d/R will become large and the value of μ that will ensure stability must be reduced. Since the stability argument is based on a linear approximation, it can be extended to higher dimensions by considering the curvature of projections of the high-dimensional constraint manifold.

Experimental Arrangement

The flat plate used for all of the experiments described next was based on the work described by Reynolds and Saric.[14] The plate was 1.52 m $\times$ 0.23 m $\times$ 1.51 cm, with an elliptical leading edge of 0.3 m in length, which was carefully machined from wood. The plate was constructed from aluminum honeycomb with 1.2-mm-thick aluminum skins. A sketch of the plate is shown in Fig. 3, which also shows the two suction panels, which were faced with laser drilled porous titanium sheet that had holes 0.1 mm in diameter randomly spaced by 1 mm, giving an open area of 0.78%, supplied by Aerospace Systems and Technology. Each panel extended 124 mm in the streamwise direction, the distance between them was also 124 mm, and the first panel started 708 mm from the leading edge. The two panels were flush mounted in the plate, care being taken to minimize the disturbance to the flow on both sides of the plate. Connections were provided to both sides of the suction blocks to aid uniform suction distribution over the whole span of the porous surface.

Each of the individual suction panels was connected to a three stage type YP3/115 British Vacuum Cleaners exhauster/blower with a three-phase 1.1 kW 220–240 V motor manufactured by D. D. Lamson plc. The speed of each pump was controlled from a personal computer via an inverter of the type SDVAG 1K5 manufactured by Brook Crompton. The experimental apparatus and pump speed control system are shown in schematic form in Fig. 1. The personal computer could produce outputs between 0 and 5 V, which, when applied to the input of the inverter, gave pump speeds between 0 and 11,500 rpm, respectively. For all of the experiments reported here the controller voltage was restricted to the range 0–2 V. A hot-wire anemometer was used to determine the flow rate in each of the main suction pipes and thereby to calibrate the flow rate at the surface of each of the suction panels against the voltage control outputs of the personal computer. The result is shown in Fig. 4.

The plate was installed in a wind tunnel having a 0.305 $\times$ 0.23 m (11 $\times$ 9 in.) working section, 1.5 m in length. The maximum flow velocity that could be produced in the working section was 20 ms^{-1}, and the associated turbulence level was about 0.8%. The natural transition position was found by hot-wire measurement to be at a distance of 0.7 m from the leading edge at a mean flow speed $U_\infty = 20$ ms^{-1}, giving a Reynolds number $Re_y = 9.3 \times 10^5$. The relatively early onset of transition is almost certainly a result of the high freestream turbulence level and background noise in the tunnel. No attempt was made to investigate the details of the transition process (evolution of Tollmien–Schlichting waves, etc.) because the techniques used for transition control are not dependent on the mechanism of the transition process itself.

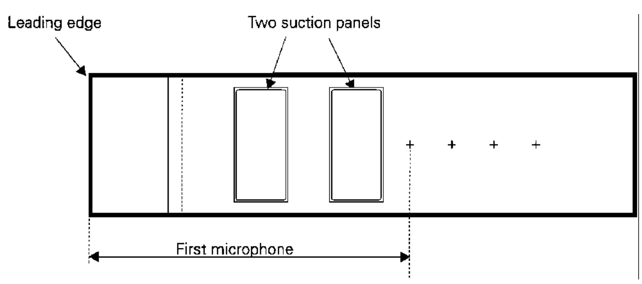

Fig. 3 Diagram of the flat plate with embedded suction panels and microphones.

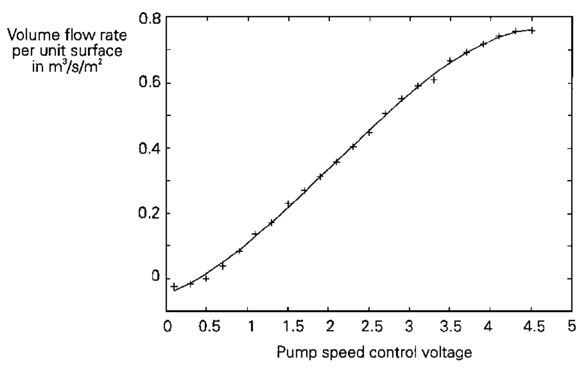

Fig. 4 Variation of pump suction flow rate and control voltage.

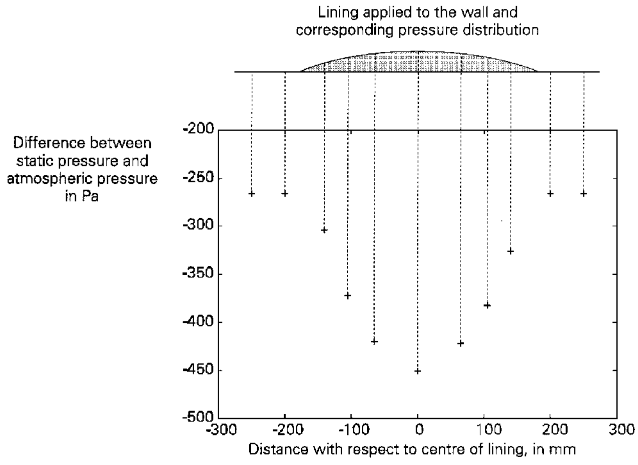

Fig. 5 Static pressure distribution induced by the wall linings.

A pressure gradient could also be applied to the plate so that the two suction panels were operating under different flow conditions, as is likely to be the case in a real application, by shaping the walls of the wind tunnel; the associated static pressure distribution is given in Fig. 5. Essentially the first suction panel was operating in a region of favorable pressure gradient, whereas the second was in a region of adverse pressure gradient.

Input/Output Measurements

The transition position was monitored using the technique described previously by Rioual et al.,[15,16] which involves the measurement of the surface pressure fluctuations in the transition region by using electret microphones mounted just below the surface of the plate. The diaphragms of the microphones communicated with the surface of the plate via small holes with diameters of 0.5 mm, giving the hole and cavity in front of the diaphragm of each microphone a Helmholtz resonance frequency in the region of 10 kHz, ensuring reliable measurements of the surface pressure fluctuations up to frequencies of about 8 kHz. The arrangement of microphones over the surface of the plate is shown in Fig. 3; specifically they were at the streamwise locations of $x_1 = 0.893$ m, $x_2 = 0.968$ m, $x_3 = 1.043$ m, and $x_4 = 1.118$ m.

These signals were high-pass filtered above 800 Hz to remove the noise from the wind-tunnel fan and then sampled at a rate of 4 kHz for a period of time T (typically 1 s). The output of the boundary layer was defined to be

$$y_k = 1 - \frac{1}{M}\sum_{m=1}^{M}\frac{\langle p(m)\rangle_k}{\langle p(m)\rangle_0} \tag{10}$$

at the kth time period, where $\langle p(m)\rangle_k$ is the rms pressure at the mth microphone at the kth time step and $\langle p(m)\rangle_0$ is its rms pressure in the absence of suction (which is premeasured using a long time average, typically $10T$).

A small value of y_k implies turbulent flow over the microphones, whereas a large value implies laminar flow over most of them; in other words, y_k is a measure of the degree to which the laminar-turbulent transition has been delayed, as long as the transition is within the array of microphones. The longer the averaging time T, the cleaner this signal will be.

A complete picture of the input/output relationship of the two-channel system was found by making a series of measurements of y for all combinations of pump speed control voltages (and thus suction rates) within a given range. The results for the plate without an applied pressure gradient are shown in Fig. 6 and in Figs. 7 and 8 for one and two pressure linings, respectively. The results in these figures clearly illustrate that the output of the system rises close to unity when large suction flow rates (high control voltages) are applied, which demonstrates that transition can be made to occur downstream of the microphone farthest from the leading edge. The results differ,

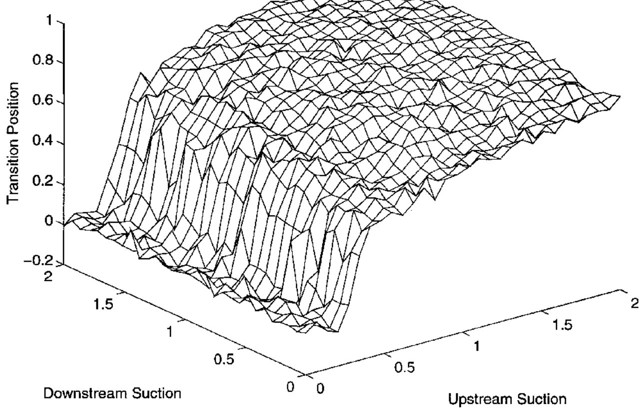

Fig. 6 Variation in microphone array output (estimated transition position) with independent upstream and downstream suction pump voltages. These measurements were made with no linings in the wind tunnel.

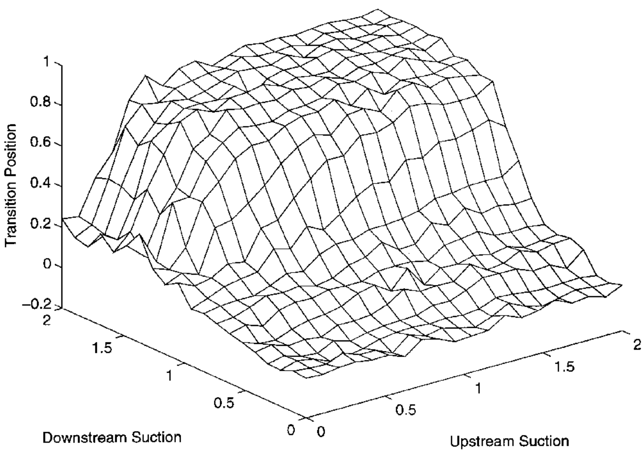

Fig. 7 Variation in microphone array output (estimated transition position) with independent upstream and downstream suction pump voltages. These measurements were made with one lining in the wind tunnel.

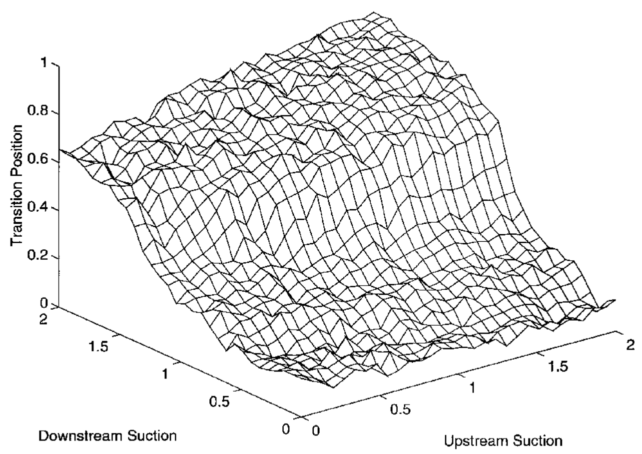

Fig. 8 Variation in microphone array output (estimated transition position) with independent upstream and downstream suction pump voltages. These measurements were made with two linings in the wind tunnel.

as one would expect, with the applied pressure gradient—basically transition is more difficult to move downstream under the action of the second suction panel, which is in a region of adverse pressure gradient. Nevertheless the two suction panels do act in combination to delay transition.

Outside a given working range the output saturates due to the finite extent of the microphone array; once the transition has moved beyond the last microphone, or before it reaches the first one, the system does not detect changes in transition position. In a practical system, care would be taken to ensure that the microphone array extended throughout the region of interest; nonetheless we used this four microphone system to demonstrate the practicalities of the situation and the capability of the control system to cope with such eventualities.

The microphone array output appears to be reduced by a small amount as the suction is increased from zero before it starts to rise. This is because there is a small pressure drop inside the tunnel due to the mean velocity of the airflow through it, and when no suction is applied, this pressure drop draws air out of the suction panels, which delays transition slightly. A small amount of suction has to be provided to balance this effect before the suction starts to have beneficial effects. The theoretical maximum value of the output is 1, which will only be reached when all of the microphones in the array are perfectly silent. In practice, of course, this will never happen as there is always some residual microphone power even when the flow over the microphones is laminar. The microphone mountings have to be carefully designed to avoid any interference to the flow, which would lead to flow-induced noise (whistling, etc.) in the laminar flow.

Results

For convenience we can make our input $\boldsymbol{u}$ be the vector of pump voltages instead of the vector of suction flow rates because, as Fig. 4 shows, they are substantially proportional, and we shall regard y as the output of the system derived from the microphone signals that characterize the position of transition. Since we are modeling $g(\boldsymbol{u})$ as a known function, we can calculate its gradient directly, but the gradient ∇e must be estimated at each iteration of the algorithm. This was done by varying the voltage applied to each pump in turn by a small amount δu and measuring the resultant change in e.

The convergence of the basic algorithm was then examined by maintaining a constant mean flow speed of $U_\infty = 9\ \mathrm{ms}^{-1}$ and requiring that the output of the system be held constant at $y = r = 0.5$. The form of the function $e(\boldsymbol{u})$ is illustrated as a contour plot in Figs. 9–11. These illustrate the successful convergence of the algorithm from three different start positions with values of $\mu = \alpha = 0.05$. These results indicate that the algorithm given by Eq. (5) is a suitable candidate for the online optimization of suction distributions.

A simple attempt to fit a circle to the zero-error constraint gives $d/R \approx \frac{1}{3}$, which suggests that the maximum permissible value of μ is about 1.5. For values of μ greater than this, therefore, the algorithm is likely to exhibit bifurcations in the region of the constrained optimum as has been demonstrated for simulated error functions.[12] To test this stability criterion with an experimentally measured error function, we applied the algorithm repeatedly with increasing values of μ. Figure 12 shows the last five values of u_1 plotted against

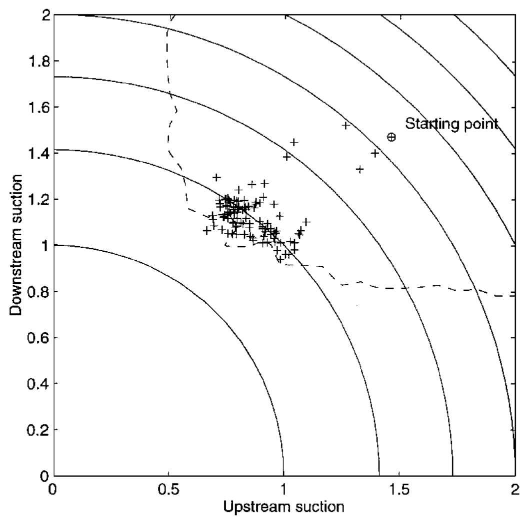

Fig. 9 Convergence of the control system under experimental conditions. The dashed line indicates the zero-error contour, the circular lines are contours of equal suction energy cost, and + indicates successive pairs of suction voltages. The constrained optimum is reached and then maintained as a steady state.

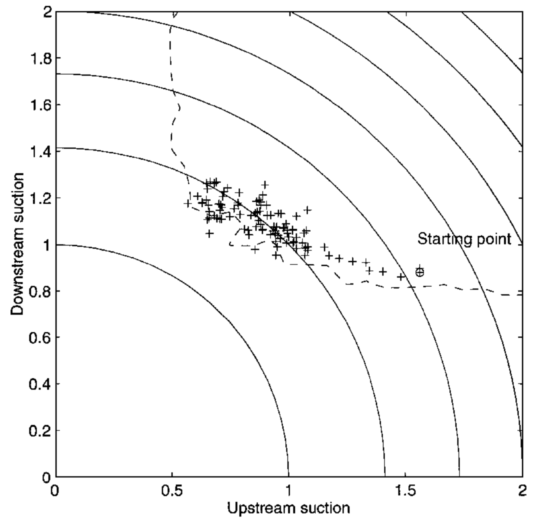

Fig. 10 Similar plot from the same experiment as that shown in Fig. 9, showing convergence from a different initial condition.

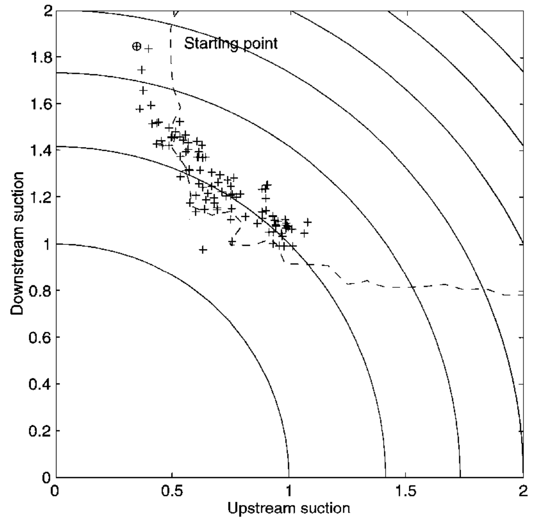

Fig. 11 Another plot similar to that in Fig. 9, showing convergence from yet another initial condition.

μ for each run. Figure 12 does not show a distinct bifurcation as the simulation results did; this may be due to noise or local variation in the curvature of the constraint contour. Nonetheless, it shows that stable convergence with reasonable immunity to noise is possible when μ is chosen to be one order of magnitude lower than that given by Eq. (9). In practice, of course, the curvature and location of the optimum would not be known in advance.

Speed of Adaptation and Convergence

In any practical implementation it will be important for the system to both converge and adapt to changes in conditions as quickly as possible. The preceding results show that with two suction panels the algorithm can reliably converge in $\mathcal{O}(50)$ iterations. Numerical simulations suggest that for a fixed convergence coefficient the number of iterations to convergence will not vary very much with the dimension of $\boldsymbol{u}$ (i.e., N, the number of suction panels).

The transition, rather than occurring suddenly, develops turbulent spots (isolated bursts of turbulence) before the onset of full-blown turbulence. To accurately estimate the rms pressure fluctuation in these regions, it is necessary to use a time period that captures a representative number of bursts. At the low speeds at which this experiment was carried out (9 ms^{-1}), this required a relatively long time average (on the order of 1 s); however, as the mean flow velocity increases, so does the spotting rate, which means that at realistic flight speeds T could be reduced. There will inevitably be a tradeoff

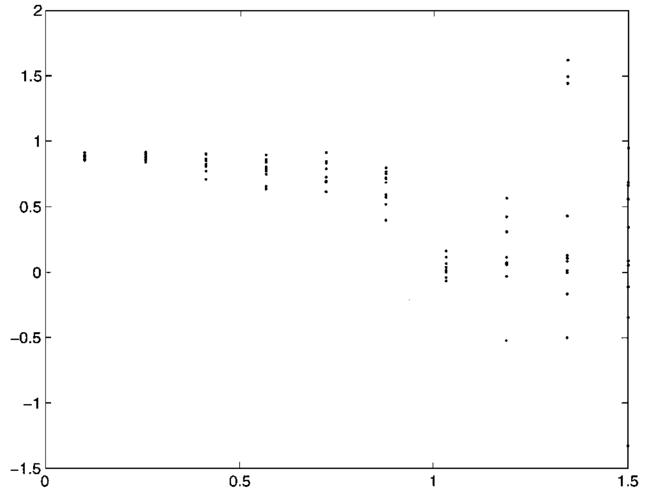

Fig. 12 Plot showing the variation in the final state of the algorithm (i.e., the last 12 values of u_1 of a 100 iteration run of the algorithm) with the convergence coefficient μ.

between μ and T; low values of T will lead to noisy estimates of e and ∇e, which in turn will require lower values of μ to ensure steady convergence. The best choices of T and μ will differ with the situation. With a greater number of microphones the statistical significance of those in the bursting region will, of course, be reduced so that T could be reduced significantly.

The finite difference estimation of ∇e takes $2NT$ s per iteration, and for large numbers of suction panels it would be desirable to find a faster method that used the values of e at successive iterations. Indeed it may, for large enough N, become desirable to deliberately restrict $|\boldsymbol{u}_k - \boldsymbol{u}_{k-1}|$, the variation in suction at each iteration, to provide fast, accurate gradient estimates. This is currently being investigated. In previous papers, such as Refs. 10 and 16, the recursive least-squares algorithm was used to estimate the gradient on the basis of a single error measurement per iteration. This has been found to be capable of producing large errors in the estimated gradient; a detailed comparison of the two methods is given in Ref. 17.

Conclusions

An algorithm has been presented that enables the automatic adjustment of the suction flow rates in a laminar flow control system. The approach taken is to adjust the suction flow rates to maintain the position of boundary-layer transition at a desired streamwise location with minimum sum of squared suction rates. This enables us to treat the overall problem of optimizing the transition position as a one-dimensional rather than multidimensional problem. The stability of the algorithm is determined by the values of the convergence parameters used and the nature of the function that describes the dependence of the position of transition on suction flow rate.

We have presented the results of an experiment wherein this algorithm was used to optimize the suction rates of two panels mounted in a flat plate in a wind tunnel with a nonzero pressure gradient, using an array of surface-mounted microphones to determine the transition position. The results show that the algorithm is capable of minimizing the suction energy needed to maintain the transition at a particular position. An investigation of the stability characteristics of the overall system has produced results in accordance with theory.

The existence of such an algorithm allows us to adopt a philosophy of on-line optimization of suction distribution for aircraft, which can be seen as an advance on the standard policy of drag reduction. Here we attempt to reduce drag only insofar as it improves the energy efficiency of the whole aircraft. In cases where, say, the geometry and, hence, the pressure gradient induce multiple minima in the optimization problem, then more modern techniques such as genetic algorithms and simulated annealing would be appropriate, and these are currently being investigated.

Acknowledgments

M. C. M. Wright is funded by the United Kingdom Engineering and Physical Sciences Research Council, and J.-L. Rioual was funded by Rolls-Royce plc. The authors are also grateful for the contributions to this work made by J.-L. Dubois and P. Saunois, both of whom were visiting students at the Institute of Sound and Vibration Research during their studies at Ecole Centrale de Paris. The flat plate and suction panel assembly used in these experiments was constructed by A. Sanger.

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